# THE SQUAREA CHALLENGE

- Area
- Square roots
- Spatial reasoning

## Getting Ready

### What You'll Need

Geoboards, 1 per student

Rubber bands

Geodot paper, page 123

Overhead Geoboard and/or geodot paper transparency (optional)

Calculators (optional)

Activity Master, page 108

### Overview

Students search to find all the different-sized squares that can be made on a Geoboard. They then investigate ways to determine the lengths of the sides of their squares. In this activity, students have the opportunity to:

- develop strategies for finding areas of squares
- learn about square roots
- apply the Pythagorean theorem
- use logical reasoning to solve a problem

Other *Super Source* activities that explore these and related concepts are:

*Glass Triangles*, page 67

*Colorful Kites*, page 72

## The Activity

### On Their Own (Part 1)

***Take the Squarea Challenge: How many different-sized squares can be made on a Geoboard?***

- Work with a partner. Make as many different-sized squares as you can on your Geoboard. Each vertex must be a peg on the board.
- Find the area of each of your squares. Let the area of the smallest possible square be 1 square unit.
- Record each square on geodot paper and label its area.
- Be ready to explain how you know you have found all possible different-sized squares that can be made.

### *Thinking and Sharing*

Invite pairs to show one of their squares and explain how they found its area. (You may want to have students use overhead materials for this.) Have them post their squares and label the areas. Be sure that each possible area is represented on the board.

Use prompts like these to promote class discussion:

- How did you go about searching for squares with different areas?
- How did you find the areas of your squares?
- Did you use more than one method for finding area? If so, describe the methods you used.
- How did you decide that you had found all the different-sized squares? How could you convince someone else that no others exist?

## On Their Own (Part 2)

**What if... *you wanted to find the lengths of the sides of each of your squares? How might you do this?***

- Working with your partner, find the lengths of the sides of each of your squares. Let the unit of measure be the horizontal distance between two consecutive pegs in the square.
- Label the side lengths on your recordings.
- Be ready to explain the method(s) you used to find the lengths of the sides of your squares.

### *Thinking and Sharing*

Refer to the posted squares and invite students to explain how they found the lengths of the sides of each square. Encourage students who used different methods to tell about the methods they used (for example, the Pythagorean theorem, square roots, estimation based on the formula for area of a square, and so on).

Use prompts like these to promote class discussion:

- How did you find the lengths of the sides of your squares?
- Did you use more than one method? If so, describe the methods you used.
- Are the measurements you found approximations or exact values? Explain.
- How is the length of the side of a square related to the area of the square?

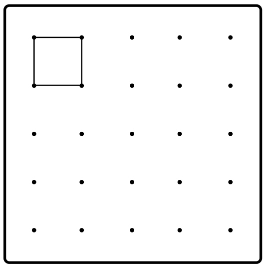

Suppose your Geoboard had an extra row and column of pegs. What would be the area of the largest square you could make on your Geoboard? What would be the area of the second-largest square you could make? What would be the lengths of the sides of these squares? Use diagrams to help support your explanations.

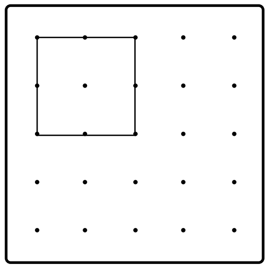
## Teacher Talk

### *Where's the Mathematics?*

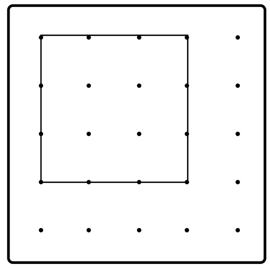
Students should find that there are eight different-sized squares that can be made on a Geoboard. The squares with areas of 1, 4, 9, and 16 square units will probably be the easiest for them to find. The squares with areas of 2, 5, 8, and 10 square units may be less obvious, as their sides are not parallel to the edges of the Geoboard.



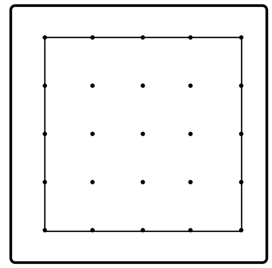
A
1 square unit



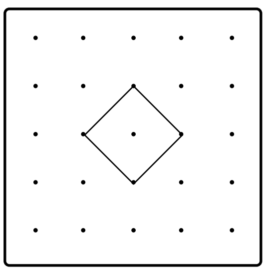
B
4 square units



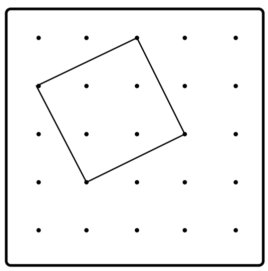
C
9 square units



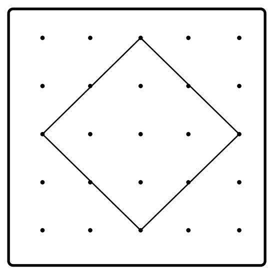
D
16 square units



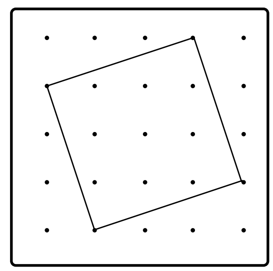
E
2 square units



F
5 square units

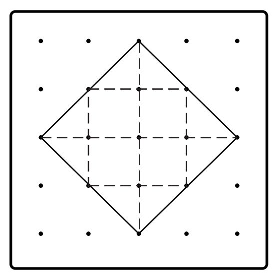


G
8 square units



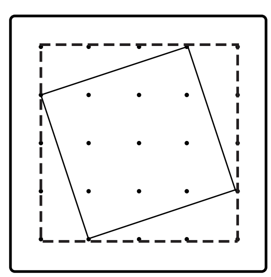
H
10 square units

Students may use a variety of methods to find the areas of their squares. For squares A through D shown above, some students may simply count the number of 1-by-1 unit squares contained in their squares, while others may use the formula for finding area of a square: Area = side x side, or $(\text{side})^2$. These methods are not easily applied to squares E through H however. To find the areas of these squares, some students may divide their squares into unit squares and parts of unit squares, and add these areas together. For example, square G can be divided into 4 unit squares, each with an area of 1 square unit, and 8 triangles, each with an area of $\frac{1}{2}$ square unit, resulting in a total of 8 square units.



Area = $4(1) + 8(\frac{1}{2}) = 8$ square units

Some students may find the area of a "tilted" square by enclosing it in a larger square whose sides are parallel to the edges of the Geoboard. They can then find the area of the larger square, and subtract the areas of the triangular regions that lie between the two squares to find the area of the tilted square. In this example, the area of the large enclosing square is 16 square units. The area of each

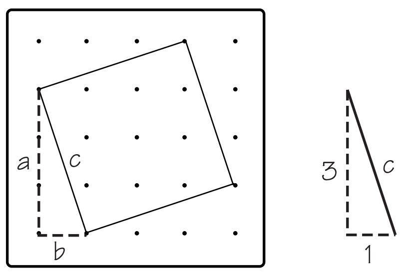


Area = $16 - 1\frac{1}{2} - 1\frac{1}{2} - 1\frac{1}{2} - 1\frac{1}{2} = 10$ square units of the triangular regions (which can be found in a number of different ways) is $1\frac{1}{2}$ square units. Thus, the area of the original square is 10 square units.

Students may describe other methods for finding area. Some may be combinations of the techniques already described, while others may be totally different approaches.

There are also a variety of techniques that students may use to find the lengths of the sides of their squares in Part 2. For squares A through D, students can simply count the number of units. The lengths of the sides of these squares are, respectively, 1, 2, 3, and 4 units. For squares E through H, the task is a bit more challenging.

Some students may visualize the side of a "tilted" square as the hypotenuse of a right triangle with legs parallel to the sides of the Geoboard. They can then use the Pythagorean theorem to find the desired length.

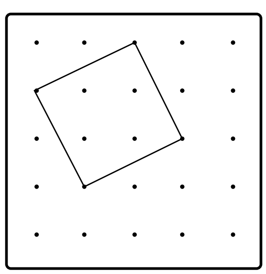



$$a^2 + b^2 = c^2$$
$$3^2 + 1^2 = c^2$$
$$9 + 1 = c^2$$
$$10 = c^2$$
$$\sqrt{10} = c$$

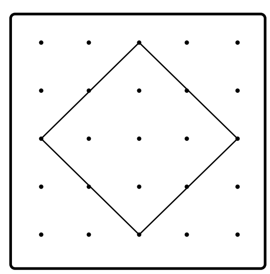
The length of the side is $\sqrt{10}$ units which is approximately 3.16 units

Others may approximate the length of a side by using the fact that the square of the side length must be the area of the square. They may reason, for example, that since square E has an area of 2 square units, the length of one of its sides must be greater than 1 and less than 2 (since the squares of these numbers are 1 and 4). Using trial-and-error on successive approximations, students may determine that the length of the side is about 1.41 units (an approximation of $\sqrt{2}$).

Students who are familiar with square roots may realize that since the area of a square is equal to the square of a side, the length of the side must be equal to the square root of the area. These students may use calculators to calculate approximations of the side lengths, or may represent the lengths using the square root symbol, $\sqrt{\phantom{x}}$. Measurements written with the square root symbol are representations of exact values, while their decimal approximations are not.



Area of square = 5 square units
Length of side = $\sqrt{5}$ units ≈ 2.24 units



Area of square = 8 square units
Length of side = $\sqrt{8}$ units ≈ 2.83 units